CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

MATHEMATICS

The Lee Fields Medal II

TIME ALLOWED: UP TO THREE HOURS

TABLES AND CALCULATORS MAY BE USED.

ANSWER <u>ALL</u> TEN QUESTIONS

1. Steven is deep in thought looking for positive whole numbers m and n that satisfy the equation
$$20m + 19n = 2020.$$
That is easy says Julie, just take $m = 101$ and $n = 0$. Of course, she's correct. Can you find another solution where $0 < m < 10$?

2. The following holds for any real numbers a, and r such that $|r| < 1$:
$$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1 - r}$$
Hence, or otherwise, express
$$0.42424242\cdots$$
as a rational number.

3. Consider a row of n tiles, each tile of which is red or blue. Suppose that a blue tile is never followed by a blue tile, so that $RBRBRR$ is allowed, but $RBBRRB$ is not.

   Let $t(n)$ be the number of allowed tilings. Find $t(1)$, and $t(2)$, and come up with a formula for $t(n)$ in terms of $t(n-1)$ and $t(n-2)$.
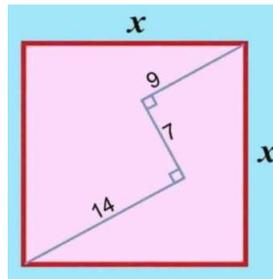
4. Suppose that m and c are constants. What is the equation of the tangent to the graph $y = mx + c$ at $x = 1$?

5. Find r

What is the radius of the smallest circle ?
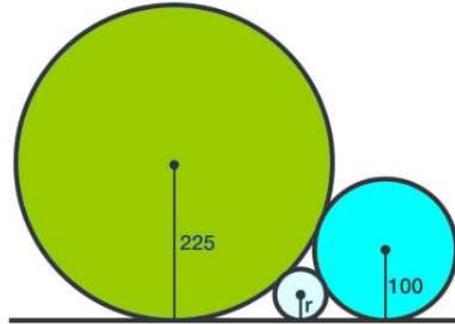
6. Find x

7. In a game show you have to choose one of three doors. One conceals a new car and the other two contain angry lions who will attack you. You choose but your chosen door is not opened immediately. Instead the presenter tells you that another door (which you have not picked), contains a lion. You then have the opportunity to change your mind. Is there any benefit to doing so? Justify your answer.

8. Write down an expression for
$$\frac{d^{2019}}{dx^{2019}} x^{2019},$$
the 2019-th derivative of x^{2019}.

9. If you expand
$$(1+x)^{2019} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_{2019} x^{2019},$$
what is the coefficient of x^2, a_2?

10. Using a 5-litre container and a 7-litre container, what is the minimum supply of water you need to measure exactly 4 litres of water?